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SQUARE SUM PRIMELABELING OF SOME STAR RELATED GRAPHS

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ABSTRACT

Square sum prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the edges with sum of the squares of the labels of the incident vertices .The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits square sum prime labeling. Here we investigate some star related graphs for square sum prime labeling.

Keywords: Graph labeling, square sum, prime labeling, prime graphs, star graph.

1. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In [5] ,we introduced the concept, square sum prime labeling and proved that some path related graphs admit this kind of labeling. In [6], [7] , [8] and [9] we extended our study and proved that the result is true for some snake graphs, some cycle related graphs, some tree graphs,jewel graph, jelly fish graph, splitting graph of star,double graph of star.Here we investigate some star related graphs for square sum prime labeling.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2.Main Results

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection

f : V(G) → {0,1,2,---, p-1} by f(v_i) = $i-1$, for every i from 1 to p and define a 1-1 mapping f_{ssp}^* : E(G) → set of natural numbers N by $f_{ssp}^*(uv)$ = f(u)^2 + f(v)^2. The induced function f_{ssp}^* is said to be a square sum prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits square sum prime labeling is called a square sum prime graph.

Theorem 2.1 Let G be the graph obtained by duplicating the apex vertex of star $K_{1,n}$by an edge. G admits square sum prime labeling.

Proof: Let G be the graph and let v_1,v_2,---,$v_{n+3}$are the vertices of G.

Here |V(G)| = n+3 and |E(G)| = n+3

Define a function f : V → {0,1,2,---,n+2 } by

f(v_i) = i-1 , i = 1,2,---,n+3

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_i v_{i+1})$ = $2i^2$-2i+1, i = 1,2,3.

$f_{ssp}^*(v_1 v_3)$ = 4.

$f_{ssp}^*(v_3 v_{i+4})$ = i^2+6i+13, i = 1,2,---,n-1.

Clearly f_{ssp}^* is an injection.

gcin of (v_1) = gcd of {$f_{ssp}^*(v_1v_2), f_{ssp}^*(v_1v_3)$ }

= gcd of {1, 4}

gcin of (v_2) = gcd of {$f_{ssp}^*(v_1v_2), f_{ssp}^*(v_2v_3)$ }

= gcd of {1, 5}

gcin of (v_3) = gcd of {$f_{ssp}^*(v_2v_3), f_{ssp}^*(v_3v_4)$ }

= gcd of {5, 13}

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square sum prime labeling.

Theorem 2.2 Let G be the graph obtained by duplicating the apex vertex of star $K_{1,n}$. G admits square sum prime labeling.

Proof: Let G be the graph and let a, b, $v_1,v_2,---,v_n$ are the vertices of G.

Here |V(G)| = n+2 and |E(G)| = 2n

Define a function f : V → {0,1,2,---,n+1 } by

$f(v_i) = i+1$, i = 1,2,---,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssp} is defined as follows

$f^*_{ssp}(a\ v_i)$ = $(i+1)^2$, i = 1,2,---,n.

$f^*_{ssp}(b\ v_i)$ = $(i+1)^2 + 1$, i = 1,2,---,n.

Clearly f^*_{ssp} is an injection.

gcin of (a) = gcd of $\{f^*_{ssp}(a\ v_1), f^*_{ssp}(a\ v_2)\}$

= gcd of {4, 9}= 1.

gcin of (v_i) = gcd of $\{f^*_{ssp}(a\ v_i), f^*_{ssp}(b\ v_i)\}$

= gcd of $\{(i+1)^2, (i+1)^2 + 1\}$

= 1, i = 1,2,---,n.

gcin of (b) = gcd of $\{f^*_{ssp}(b\ v_1), f^*_{ssp}(b\ v_3)\}$

= gcd of {5, 17}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square sum prime labeling.

Theorem 2.3 Lily graph admits square sum prime labeling.

Proof: Let G = Ll_n and let $v_1,v_2,---,v_{4n-1}$ are the vertices of G.

Here |V(G)| = 4n-1 and |E(G)| = 4n-2

Define a function f : V → {0,1,2,-----------------,4n-2 } by

$f(v_i) = i-1$, i = 1,2,------,4n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssp} is defined as follows

$f_{ssp}^{*}(v_i v_{i+1})$ = $2i^2$-2i+1, i = 1,2,---,2n-2

$f_{ssp}^{*}(v_n v_{3n+i-1})$ = $(3n+i-2)^2 + (n-1)^2$, i = 1,2,---,n.

$f_{ssp}^{*}(v_n v_{2n+i-1})$ = $(2n+i-2)^2 + (n-1)^2$, i = 1,2,---,n.

Clearly f_{ssp}^{*} is an injection.

gcin of (v_{i+1}) = gcd of $\{f_{ssp}^{*}(v_i v_{i+1}), f_{ssp}^{*}(v_{i+1} v_{i+2})\}$

= gcd of $\{2i^2-2i+1, 2i^2+2i+1\}$

= gcd o $\{4i, 2i^2-2i+1\}$

=1, i = 1,2,---,2n-3.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence L_n, admits square sum prime labeling.

Theorem 2.4 Let G_1 be the first copy of star $K_{1,n}$and G_2 be the second copy of star $K_{1,n}$. Let a be the central vertex and let v_1,v_2,---,v_n are the pendant vertices of G_1.Let b be the central vertex and let u_1, u_2,---,u_n are the pendant vertices of G_2 .Let G be the graph obtained by replacing the vertices u_i, v_i by w_i and joining a to w_i and b to w_i , for every i. G admits square sum prime labeling.

Proof: Let G be the graph and let a, b, w_1,w_2,---,$w_n$are the vertices of G.

Here |V(G)| = n+2 and |E(G)| = 2n

Define a function f : V → {0,1,2,---,n+1 } by

f(w_i) = i+1 , i = 1,2,---,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^{*} is defined as follows

$f_{ssp}^{*}(a\, w_i)$ = $(i+1)^2$, i = 1,2,---,n.

$f_{ssp}^{*}(b\, w_i)$ = $(i+1)^2+1$, i = 1,2,---,n.

Clearly f_{ssp}^{*} is an injection.

gcin of (w_i) = gcd of $\{f_{ssp}^{*}(a\, w_i), f_{ssp}^{*}(b\, w_i)\}$

= gcd of $\{(i+1)^2, (i+1)^2+1\}$

= 1, i = 1,2,---,n.

gcin of (a) = gcd of $\{f_{ssp}^{*}(a\, w_1), f_{ssp}^{*}(a\, w_2)\}$

= gcd of {4, 9}= 1.

gcin of (b) = gcd of $\{f_{ssp}^*(b\ w_1), f_{ssp}^*(b\ w_3)\ \}$

= gcd of {5, 17}=1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square sum prime labeling.

Theorem 2.5 Strong Double graph of star graph$K_{1,n}$(n is a natural number greater than 3) admits square sum prime labeling.

Proof: Let G =S{D($K_{1,n}$)} and let a,b,v_1,v_2,---,v_n,u_1,u_2,---,$u_n$are the vertices of G.

Here |V(G)| = 2n+2 and |E(G)| = 5n+1

Define a function f : V → {0,1,2,---,2n-1 } by

$f(v_i)$ = i+1 , i = 1,2,---,n

$f(u_i)$ = n+i+1 , i = 1,2,---,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(a\ v_i)$ = $(i+1)^2$, i = 1,2,---,n.

$f_{ssp}^*(a\ u_i)$ = $(n+i+1)^2$, i = 1,2,---,n.

$f_{ssp}^*(b\ v_i)$ = $(i+1)^2 + 1$, i = 1,2,---,n.

$f_{ssp}^*(b\ u_i)$ = $(n+i+1)^2 + 1$, i = 1,2,---,n.

$f_{ssp}^*(u_i v_i)$ = $(n+i+1)^2 + (i+1)^2$, i = 1,2,---,n.

$f_{ssp}^*(a\ b)$ = 1.

Clearly f_{ssp}^* is an injection.

gcin of (a) = gcd of $\{f_{ssp}^*(a\ v_1), f_{ssp}^*(a\ v_2)\ \}$

= gcd of {4, 9}= 1.

gcin of (v_i) = gcd of $\{f_{ssp}^*(a\ v_i), f_{ssp}^*(b\ v_i)\ \}$

= gcd of { $(i+1)^2$, $(i+1)^2 + 1$}

= 1, i = 1,2,---,n.

gcin of (b) = gcd of $\{f_{ssp}^*(b\ v_1), f_{ssp}^*(b\ v_2)\ \}$

= gcd of {3, 8}= 1.

gcin of (u_i) = gcd of $\{f_{ssp}^*(a\ u_i), f_{ssp}^*(b\ u_i)\ \}$

= gcd of { $(n+i+1)^2$, $(n+i+1)^2 + 1$}

= 1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence S{D($K_{1,n}$)} , admits square sum prime labeling.

Theorem 2.6 Let G be the graph obtained by joining the apex vertex of star $K_{1,n}$to the pendant vertices of path P_n. G is called double coconut tree graph and is denoted by DCT(n,n,n). G admits square sum prime labeling.

Proof: Let G =DCT(n,n,n) and let $v_1,v_2,---,v_{3n}$are the vertices of G.

Here |V(G)| = 3n and |E(G)| = 3n-1

Define a function f : V → {0,1,2,---,3n-1 } by

$f(v_i)$ = i-1 , i = 1,2,---,3n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_{n+i-1}v_{n+i})$ = $(n+i-1)^2 + (n+i-2)^2$, i = 1,2,---,n+1

$f_{ssp}^*(v_{n+1}v_i)$ = $n^2 + (i-1)^2$, i = 1,2,---,n-1.

$f_{ssp}^*(v_{2n}v_{2n+i+1})$ = $(2n+i)^2 + (2n-1)^2$, i = 1,2,---,n-1.

Clearly f_{sdp}^* is an injection.

gcin of (v_{n+i}) = gcd of {$f_{ssp}^*(v_{n+i-1}v_{n+i}), f_{ssp}^*(v_{n+i}v_{n+i+1})$ }

= gcd of {$2(n+i)^2-6(n+i)+5$, $2(n+i)^2-2(n+i)+1$}

= 1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence DCT(n,n,n), admits square sum prime labeling.

Theorem 2.7 Let G be the graph obtained by joining the apex vertex of star $K_{1,n}$to any one vertex of cycle C_n. G admits square sum prime labeling.

Proof: Let G be the graphand let $v_1,v_2,---,v_{2n}$are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n

Define a function f : V → {0,1,2,---,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,---,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ssp}^* is defined as follows

$f_{ssp}^*(v_iv_{i+1})$ = $2i^2-2i+1$, i = 1,2,---,n.

$f^*_{ssp}(v_n v_1)$ = $(n-1)^2$.

$f^*_{ssp}(v_n v_{n+i+1})$ = $(n+i)^2 + (n-1)^2$, i = 1,2,---,n-1.

Clearly f^*_{ssp} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{ssp}(v_i v_{i+1}), f^*_{ssp}(v_{i+1} v_{i+2})\}$

= gcd of $\{2i^2-2i+1, 2i^2+2i+1\}$

= gcd o $\{4i, 2i^2-2i+1\}$

= 1, i = 1,2,---,n-1.

gcin of (v_1) = gcd of $\{f^*_{ssp}(v_1 v_2), f^*_{ssp}(v_1 v_n)\}$

= gcd of $\{1, (n-1)^2\}$ = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square sum prime labeling.

Theorem 2.8 Let G be the graph obtained by joining the apex vertex of star $K_{1,3}$to all vertices of path P_n. G admits square sum prime labeling.

Proof: Let G be the graph and let v_1, v_2,---,$v_{4n}$are the vertices of G.

Here |V(G)| = 4n and |E(G)| = 4n-1

Define a function f : V → {0,1,2,---,4n-1 } by

$f(v_i)$ = i-1 , i = 1,2,---,4n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssp} is defined as follows

$f^*_{ssp}(v_{4i-3} v_{4i-2})$ = $32i^2-56i+25$, i = 1,2,---,n.

$f^*_{ssp}(v_{4i-3} v_{4i-1})$ = $32i^2-48i+20$, i = 1,2,---,n.

$f^*_{ssp}(v_{4i-3} v_{4i})$ = $32i^2-40i+17$, i = 1,2,---,n.

$f^*_{ssp}(v_{4i-3} v_{4i+1})$ = $32i^2-16i+16$, i = 1,2,---,n-1.

Clearly f^*_{ssp} is an injection.

gcin of (v_{4i-3}) = gcd of $\{f^*_{ssp}(v_{4i-3} v_{4i-2}), f^*_{ssp}(v_{4i-3} v_{4i-1}), f^*_{ssp}(v_{4i-3} v_{4i})\}$

= 1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits square sum prime labeling.

REFERENCES

[1]. Apostol. Tom M, Introduction to Analytic Number Theory, Narosa, (1998).

[2]. F Harary, Graph Theory, Addison-Wesley,Reading, Mass, (1972).

[3]. Joseph A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics(2015), #DS6, Pages 1 – 389.

[4]. T K Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala 2000.

[5]. Sunoj B S, Mathew Varkey T K, Square Sum Prime Labeling of Some Path Related Graphs, International Journal of Application or Innovation in Engineering & Management, Volume 6, Issue 7, July 2017, pp 25-28.

[6]. Sunoj B S, Mathew Varkey T K, Square Sum Prime Labeling of Some Cycle Related Graphs, International Journal of Advanced Research in Engineering and Applied Sciences, Volume 6, Issue 7, July 2017, pp 12-17.

[7]. Sunoj B S, Mathew Varkey T K, Square Sum Prime Labeling of Some Tree Graphs, International Journal of Information Movement, Volume 2, Issue 3, July 2017, pp 164-167.

[8]. Sunoj B S, Mathew Varkey T K, A Note on Square Sum Prime Labeling,International Journal of Engineering, Science and Mathematics, Volume 6, Issue 4, August 2017, pp 90-95.

[9]. Sunoj B S, Mathew Varkey T K, Square Difference Prime Labeling of Some Snake Graphs, International Journal of Scientific Research, Volume 6, Issue 9, September 2017, pp 598-600.